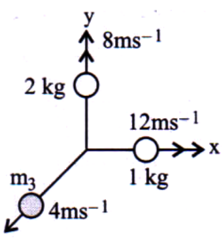
WEEKLY TEST TYJ-02 TEST 11 RAJPUR ROAD
SOLUTION Date 13-10-2019

[PHYSICS]

1.

The situation of the problem is as shown in the figure. According to law of conservation of linear momentum



$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

$$\therefore \quad \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$$

Here,

$$\vec{p}_1 = (1\text{kg})\,(12\text{ ms}^{-1})\hat{i} = 12\,\hat{i}\text{ kg ms}^{-1}$$

$$\vec{p}_2 = (2\text{ kg})\,(8\text{ ms}^{-1})\hat{j} = 16\hat{j}\text{ kg ms}^{-1}$$

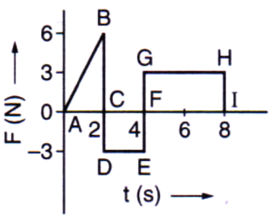
$$\therefore \quad \vec{p}_3 = -(12\,\hat{i} + 16\hat{j})\text{ kg ms}^{-1}$$

The magnitude of $\vec{p}_3$ is :

$$p_3 = \sqrt{(12)^2 + (16)^2} = 20\text{ kg ms}^{-1}$$

$$\therefore \quad m_3 = \frac{p_3}{v_3} = \frac{20\text{ kg ms}^{-1}}{4\text{ ms}^{-1}} = 5\text{kg}$$

2.



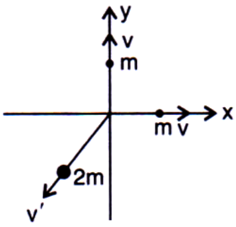
Change in momentum = Area under *F-t* graph in that interval

= Area of ΔABC – Area of rectangle *CDEF* + Area of rectangle *FGHI*

$$= \frac{1}{2}\times 2\times 6 - 3\times 2 + 4\times 3 = 12\text{ N s}$$

3.

Let $\vec{v'}$ be velocity of third piece of mass $2m$. Initial momentum, $\vec{P_i} = 0$ (As the body is at rest). Final momentum,



$\vec{P_f} = mv\hat{i} + mv\hat{j} + 2m\vec{v'}$

According to law of conservation of momentum

$\vec{p_i} = \vec{p_f}$

$0 = mv\hat{i} + mv\,\hat{j} + 2m\vec{v'}$

$\vec{v'} = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$

The magnitude of $\vec{v'}$ is

$v' = \sqrt{\left(-\frac{v}{2}\right)^2 + \left(-\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

Total kinetic energy generated due to explosion

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2 = mv^2 + \frac{mv^2}{2}$

$= \frac{3}{2}mv^2$

4.

Given that,

$\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ and $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

Hence, $v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$\therefore\ P = \vec{F}\cdot\vec{v} = 2t\cdot t^2 + 3t^2\cdot t^3 = 2t^3 + 3t^5$

5.

Here, $m_1 = m, m_2 = 2m$

$u_1 = 2$ m/s, $u_2 = 0$

Coefficient of restitution, $e = 0.5$

Let v_1 and v_2 be their respective velocities after collision.

Applying the law of conservation of linear momentum, we get,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$\therefore$ $$m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$$

or $$2m = mv_1 + 2mv_2$$

or $$2 = (v_1 + 2v_2) \quad \ldots\text{(i)}$$

By definition of coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

or $$e(u_1 - u_2) = (v_2 - v_1)$$

$$0.5(2 - 0) = (v_2 - v_1)$$

$$1 = v_2 - v_1 \quad \ldots\text{(ii)}$$

Solving equations (i) and (ii), we get,

$$v_1 = 0 \text{ m/s}, v_2 = 1 \text{ m/s}$$

6.

According to conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v,$$

where v is common velocity of the two bodies.

$m_1 = 0.1\,\text{kg}$, $m_2 = 0.4\,\text{kg}$

$v_1 = 1\,\text{m/s}$, $v_2 = -0.1\,\text{m/s}$

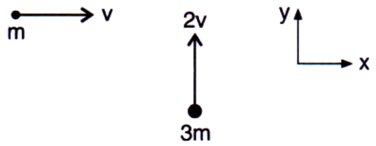
$\therefore\ 0.1\times1 + 0.4\times(-0.1) = (0.1 + 0.4)v$

or $0.1 - 0.04 = 0.5v,$

$$v = \frac{0.06}{0.5} = 0.12\ \text{m/s}.$$

Hence, distance covered = $0.12 \times 10 = 1.2$ m

7.



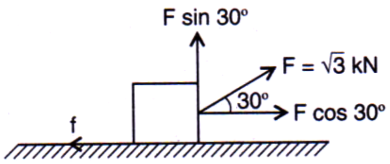
According to conservation of momentum, we get

$$mv\hat{i} + (3m)2v\hat{j} = (m + 3m)v'$$

where v' is the final velocity after collision

$$v' = \frac{1}{4}v\hat{i} + \frac{6}{4}v\hat{j} = \frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}.$$

8.



The component of applied force F in the direction of motion is $F\cos 30°$.

The work done by the applied force is,

$$W = (F\cos 30°)S = \sqrt{3}\times10^3\times\frac{\sqrt{3}}{2}\times10\,\text{J}$$

$$= 15\times10^3\ \text{J} = 15\ \text{kJ}.$$

9.

Mass of water falling/second = 15 kg, $h = 60$ m

$g = 10\,\text{m/s}^2$, loss =10% , *i.e.*,90% is used

Power generated $= 15\times10\times60\times0.9 = 8100$ W

$= 8.1$ kW

10. $$mv = Mv' \quad \text{or} \quad v' = \left(\frac{m}{M}\right)v$$

Total KE of the bullet and the gun $= \frac{1}{2}mv^2 + \frac{1}{2}Mv'^2$

$$\text{Total KE} = \frac{1}{2}mv^2 + \frac{1}{2}M \cdot \frac{m^2}{M^2}v^2$$

$$\text{Total KE} = \frac{1}{2}mv^2\left[1 + \frac{m}{M}\right]$$

or $$1.05 \times 1000\ \text{J} = \left[\frac{1}{2} \times 0.2\right]\left[1 + \frac{0.2}{4}\right]v^2$$

or $$v^2 = \frac{4 \times 1.05 \times 1000}{0.1 \times 4.2} = (100)^2;$$

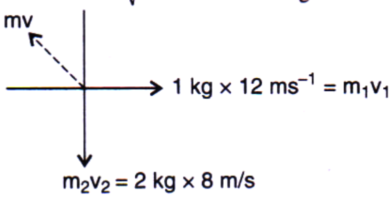
$\therefore$ $$v = 100\ \text{ms}^{-1}$$

11.

When an explosion breaks a rock, by the law of conservation of momentum, initial momentum which is zero, is equal to total momentum of three pieces.

Total momentum of the two pieces 1 kg and 2 kg

$$= \sqrt{12^2 + 16^2} = 20\ \text{kg m s}^{-1}$$



The third piece has the same momentum and in the direction opposite to the resultant of these two momenta.

$\therefore$ Momentum of the third piece $= 20\ \text{kg ms}^{-1}$;

Velocity $= 4\ \text{ms}^{-1}$

$\therefore$Mass of the 3rd piece $= \frac{mv}{v} = \frac{20}{4} = 5$ kg.

[CHEMISTRY]

16. (b)
17. (b)
18.

Higher the number of CH_3 groups on a C-atom more the $+I$ effect but on benzene or unsaturated centre, it is hyperconjugation that dominates.

19.

3Cl-atoms (E.W.G.) decrease the –ve charge to a very high extent than benzene ring. CH_3 increases electron density on $\bar{C}$ part.

20.

Acid strength is as follows

$$RCOOH > HC\equiv CH > NH_3 > RH$$

Hence, the basicity in increasing order should be reverse for conjugated bases.

$$RCOO^- < HC\equiv C^- < \bar{N}H_2 < R^-$$

21. (b)
22. (b)
23.

Because of very high $-I$ effect NO_2 helps in the release of H^+ from —COOH group, most easily.

24.

More the number of alkyl groups closer to —O—H group, more is the electron density of O-atom and more is the basic nature.

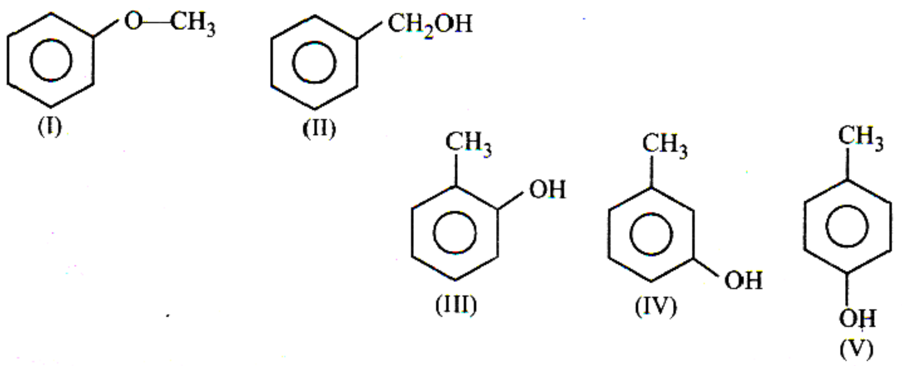
25.

Due to its **$-I$ effect**, NO_2 is decreasing the –ve charge on CH_2.

26. (a)
27. (a)
28.

(I) has 6 hyperconjugation structures and one resonance structure due to + M effect of O-atom.
(II) has 3 hyperconjugation structures and one resonance structure.
(III) has 5 hyperconjugation structures.
(IV) has 2 hyperconjugation structures.

29. (a)
30.



Compound (II) is phenylmethanol a phenyl substituted alcohol. It is studied under aliphatic compounds and not aromatic. Hence, **four** aromatic isomers.

[MATHEMATICS]

31. (b) Three letters can be posted in 4 letter boxes in $4^3 = 64$ ways but it consists the 4 ways that all letters may be posted in same box. Hence required ways = 60.

32. (b) The word ARRANGE, has AA, RR, NGE letters, that is two A' s, two R's and N, G, E one each.

$\therefore$ The total number of arrangements

$= \frac{7!}{2!2!1!1!1!} = 1260$

But, the number of arrangements in which both RR are together as one unit = $\frac{6!}{2!1!1!1!1!} = 360$

$\therefore$ The number of arrangements in which both RR do not come together = 1260 – 360 = 900.

33. (b) The number of ways can be deduce as follows :

1 woman and 4 men $= {}^4C_1 \times {}^6C_4 = 60$

2 women and 3 men $= {}^4C_2 \times {}^6C_3 = 120$

3 women and 2 men $= {}^4C_3 \times {}^6C_2 = 60$

4 women and 1 man $= {}^4C_4 \times {}^6C_1 = 6$

Required number of ways $= 60 + 120 + 60 + 6 = 246$.

34. (b) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.

Therefore the required number of ways in which the car can be filled is ${}^5C_2 \times {}^2C_1 = 20$.

35. (c) We have got $2P^s, 2R^s, 3O^s, 1I, 1T, 1N$ *i.e.* 6 types of letters. We have to form words of 4 letters. We consider four cases

(i) All 4 different : Selection ${}^6C_4 = 15$

Arrangement $= 15 . 4! = 15 \times 25 = 360$

(ii) Two different and two alike :

P^s, R^s and O^s in ${}^3C_1 = 3$ ways. Having chosen one pair we have to choose 2 different letters out of the remaining 5 different letters in ${}^5C_2 = 10$ ways. Hence the number of selections is $10 \times 3 = 30$. Each of the above 30 selections has 4 letters out of which 2 are alike and they can be arranged in $\frac{4!}{2!} = 12$ ways.

Hence number of arrangements is $12 \times 30 = 360$.

(iii) 2 like of one kind and 2 of other :

Out of these sets of three like letters we can choose 2 sets in ${}^3C_2 = 3$ ways. Each such selection will consist of 4 letters out of which 2 are alike of one kind, 2 of the other. They can be arranged in $\frac{4!}{2!\,2!} = 6$ ways.

Hence the number of arrangements is $3 \times 6 = 18$.

(iv) 3 alike and 1 different :

There is only one set consisting of 3 like letters and it can be chosen in 1 way. The remaining one letter can be chosen out of the remaining 5 types of letters in 5 ways.

Hence the number of selection $= 5 \times 1$. Each consists of 4 letters out of which 3 are alike and each of them can be arranged in $\frac{4!}{3!} = 4$ ways.

Hence the number of arrangements is $5 \times 4 = 20$.

From (i), (ii), (iii) and (iv), we get

Number of selections $= 15 + 30 + 3 + 5 = 53$

Number of arrangements

$= 360 + 360 + 18 + 20 = 758$.

36. **Two are already selected, so we have to select only 9. Four are excluded and two already selected, so we have to select out of 22 – 6 = 16. The number of ways is ${}^{16}C_9$.**

37. **(c) Here, we have 1M, 41's,4 S's and P's**
∴ Total number of selections

38 **(a) The required number of ways =**
(10 + 1) (9 + 1) (7 +1) – 1 = 879.

39. (b) $\frac{(n+1)(n+2)}{2} = 45$ or $n^2 + 3n - 88 = 0 \Rightarrow n = 8$..

40. (b) $(2+\sqrt{2})^4 = (\sqrt{2})^4(\sqrt{2}+1)^4$

$= 4[{}^4C_0 + {}^4C_1(\sqrt{2}) + {}^4C_2(\sqrt{2})^2 + {}^4C_3(\sqrt{2})^3 + {}^4C_4(\sqrt{2})^4]$

$= 4\left[1 + 4\sqrt{2} + \frac{4.3}{2}.2 + \frac{4.3.2}{1.2.3}.2\sqrt{2} + 4\right]$

$= 4[1 + 4\sqrt{2} + 12 + 8\sqrt{2} + 4] \;=\; 4[17 + 12\sqrt{2}]$

$= 4[17 + (\simeq 17)] \;=\; 4[34] = 136$.

41. (b) $5^{99} = (5)(5^2)^{49} = 5(25)^{49} = 5(26-1)^{49}$

$= 5 \times (26) \times (\text{Positive terms}) - 5$, So when it is divided by 13 it gives the remainder – 5 or (13 – 5) *i.e.*, 8.

42. (b) $(x+a)^n+(x-a)^n=2\ [x^n+{}^nC_2x^{n-2}a^2+{}^nC_4x^{n-4}a^4+$
${}^nC_6x^{n-6}a^6+.......]$

Here, $n=6, x=\sqrt{2}, a=1$; ${}^6C_2=15, {}^6C_4=15, {}^6C_6=1$

$\therefore (\sqrt{2}+1)^6(\sqrt{2}-1)^6=2[(\sqrt{2})^6+15.(\sqrt{2})^4.1$
$+15(\sqrt{2})^2.1+1.1]$

$=2[8+15\times4+15\times2+1]=198$

43. (b) We have $(1+x^2)^5(1+x)^4$

$=({}^5C_0+{}^5C_1x^2+{}^5C_2x^4+...)\ ({}^4C_0+{}^4C_1x+{}^4C_2x^2+{}^4C_3x^3+{}^4C_4x^4)$

So coefficient of x^5 in $[(1+x^2)^5(1+x)^4]$

$=\ {}^5C_2.{}^4C_1+{}^4C_3.{}^5C_1=60.$

44. (d) ${}^{18}C_{2r+3}={}^{18}C_{r-3}\Rightarrow 2r+3+r-3=18\Rightarrow r=6$

45. (c) The general term in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is $T_{r+1}={}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)x^{18-3r}$ ......(i)

Now, the coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ ......(ii)

= Sum of the coefficient of the terms x^0, x^{-1} and x^{-3} in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

For x^0 in (i) above, $18-3r=0\Rightarrow r=6$. For x^{-1} in (i) above, there exists no value of r and hence no such term exists. For x^{-3} in (i), $18-3r=-3\Rightarrow r=7$

$\therefore$ For term independent of x, in (ii) the coefficient

$=1\times{}^9C_6(-1)^6\left(\frac{3}{2}\right)^{9-6}\left(\frac{1}{3}\right)^6+2\times{}^9C_7(-1)^7\left(\frac{3}{2}\right)^{9-7}\left(\frac{1}{3}\right)^7$

$=\frac{9.8.7}{1.2.3}.\frac{3^3}{2^3}.\frac{1}{3^6}+2\frac{9.8}{1.2}(-1)\frac{3^2}{2^2}.\frac{1}{3^7}=\frac{7}{18}-\frac{2}{27}=\frac{17}{54}.$